Appendix: Alternative Approaches to the Decomposition of the Trading Gains in the Törnqvist Case

In order to correct for the homogeneity shortcomings of the original Diewert and Morrison approach, Kohli (2003, 2004a) proposed to rewrite GDP function (1) as a function of the terms of trade h_t and of the relative price of exports to nontraded goods ($p_{X,t}/p_{N,t}$). The GDP contribution of a change in the terms of trade, holding $p_{X,t}/p_{N,t}$ constant, is then given by the following expression, which is exact if the true GDP function is Translog:

$$\begin{aligned} H_{X,t,t-1} &\equiv \left(\frac{h_t}{h_{t-1}}\right)^{\bar{s}_{M,t}} \\ &= \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{M,t}} \\ &\cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}}. \end{aligned} \tag{A1}$$

This approach is essentially the Törnqvist equivalent of deflating the trade account by the price of exports when computing real GDI and deriving the trading gains in the Laspeyres case, one of the suggestions of the SNA.

(A1) is clearly homogeneous of degree zero in prices as required. Moreover, there is no difference between $DMA_{t,t-1}$ and $H_{X,t,t-1}$ if trade is balanced. This is not the end of the story, however, since one must also consider the gain or loss resulting from a change in $p_{X,t}/p_{N,t}$, holding the terms of trade constant, when trade is not balanced:[1]

$$\begin{aligned} E_{X,t,t-1} &\equiv \left(\frac{p_{X,t}/p_{N,t}}{p_{X,t-1}/p_{N,t-1}}\right)^{\bar{s}_{X,t}-\bar{s}_{M,t}} \\ &= \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}-\bar{s}_{M,t}} \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} \end{aligned} \tag{A2}$$

This term is homogeneous of degree zero in prices as well. It is noteworthy that:

$$H_{X,t,t-1} \cdot E_{X,t,t-1} = G_{t,t-1}. \tag{A3}$$

That is, this approach leads to the same estimate of the trading gains as the approach of Section 2 above. What differs is the decomposition between the terms-of-trade

1 An equiproportional increase in the prices of exports and non-traded goods, holding other things (thus including h_t) constant, has a positive impact on real income in case of a trade surplus, negative in case of a deficit.

component and the relative–price component: $H_{t,t-1}$ differs from $H_{X,t,t-1}$ since it measures the terms-of-trade effect holding $p_{T,t}/p_{N,t}$ constant, as opposed to $p_{X,t}/p_{N,t}$.

The complete decomposition of nominal GDP thus becomes:

$$\begin{aligned}\Pi_{t,t-1} &= P_{N,t,t-1} \cdot H_{X,t,t-1} \\ &\quad \cdot E_{X,t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1} \\ &= P_{N,t,t-1} \cdot G_{t,t-1} \cdot X_{t,t-1} \cdot R_{t,t-1}.\end{aligned} \tag{A4}$$

It would have been possible, of course, to use the price of imports, rather than exports, to deflate the trade account as it is so often done in practice. From a theoretical viewpoint, the choice between the two indices is perfectly arbitrary and it makes no difference as to the ultimate estimate of the trading gains: only the split between the terms-of-trade effect and the relative-price effect is affected. Nonetheless, for the sake of completeness, we will also present this case in Törnqvist form. This amounts to switching the role of the prices of imports and exports, thus replacing expression (A1)-(A2) by the following:

$$\begin{aligned}H_{M,t,t-1} &\equiv \left(\frac{h_t}{h_{t-1}}\right)^{\bar{s}_{X,t}} \\ &= \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{X,t}},\end{aligned} \tag{A5}$$

$$\begin{aligned}E_{M,t,t-1} &\equiv \left(\frac{p_{M,t}/p_{N,t}}{p_{M,t-1}/p_{N,t-1}}\right)^{\bar{s}_{X,t}-\bar{s}_{M,t}} \\ &= \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{\bar{s}_{X,t}-\bar{s}_{M,t}} \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})}\end{aligned} \tag{A6}$$

One then obtains:

$$\begin{aligned}H_{M,t,t-1} \cdot E_{M,t,t-1} &= \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \\ &\quad \cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}} \\ &\quad \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} \\ &= G_{t,t-1}\end{aligned} \tag{A7}$$

Thus, $G_{t,t-1}$, the trading-gain index, is once again the same: only the decomposition differs since the terms-of-trade effect has been defined differently, not holding the same relative price constant, and consequently $E_{M,t,t-1}$ does not relate to the same relative price as $E_{t,t-1}$ or $E_{X,t,t-1}$ either.

Yet a third possibility, the one suggested by EuroStat among others, would be to use an arithmetic average of the price of imports and exports ($P_{A,t} \equiv \frac{1}{2}p_{X,t} + \frac{1}{2}p_{M,t}$) as the deflator of the trade account. The terms-of-trade effect would then be as follows:

$$H_{A,t,t-1} \equiv \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}} \cdot \left(\frac{p_{A,t}}{p_{A,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} \tag{A8}$$

The residual, relative-price effect would then be:

$$E_{A,t,t-1} \equiv \left(\frac{p_{A,t}}{p_{A,t-1}}\right)^{(\bar{s}_{X,t}-\bar{s}_{M,t})} \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} \tag{A9}$$

and one would find once again that:

$$\begin{aligned} H_{A,t,t-1} \cdot E_{A,t,t-1} = & \left(\frac{p_{X,t}}{p_{X,t-1}}\right)^{\bar{s}_{X,t}} \\ & \cdot \left(\frac{p_{M,t}}{p_{M,t-1}}\right)^{-\bar{s}_{M,t}} \\ & \cdot \left(\frac{p_{N,t}}{p_{N,t-1}}\right)^{-(\bar{s}_{X,t}-\bar{s}_{M,t})} \\ = & \quad G_{t,t-1}. \end{aligned} \tag{A10}$$

Thus, all three approaches reviewed here come up with the same measure of the trading gains as the approach adopted in the main text, i.e. (16), and which, as indicated by (19), can be conveniently derived as the ratio of the GDP price deflator to the price of domestic final expenditures. What differs, however, is the decomposition of these trading gains between the terms-of-trade effect and the relative price effect, since that relative price itself is defined differently in all four cases. In our opinion, the approach used in the main text is to be preferred for at least two reasons. First, unlike the first two approaches reviewed here, it treats import and export prices symmetrically. Second, and most decisively, $E_{t,t-1}$, the relative price term as defined by (15), has an important economic interpretation being the real exchange rate.

It important to note that national statistical agencies, with the notable exceptions of the BEA and Statistics Canada, typically only report the Laspeyres equivalents of $H_{t,t-1}$, $H_{X,t,t-1}$, $H_{M,t,t-1}$, or $H_{A,t,t-1}$, thereby entirely neglecting the accompanying, relative-price effects. Unless trade happens to be balanced (an event with probability zero) or the relative prices are systematically constant (equally unlikely), one must conclude that the trading gains estimates these agencies publish are incomplete and misnamed (at best, they are estimates of the terms-of-trade effects).

We conclude on a somewhat ironic note. Looking at (15) and comparing it with (A9), the resemblance is striking: the measure of the terms-of-trade effect that we recommend could also be interpreted, in the Törnqvist case, as the trading-gains measure that EuroStat favours, as long as $p_{A,t} = p_{T,t}$. That is, whereas EuroStat offers no theoretical justification when writing that “in circumstances in which there is uncertainty about the choice of deflator an average of the imports and the export price indices is likely to provide a suitable deflator" (European Commission, 2013:302), it turns out that $H_{A,t,t-1}$ is exact if the underlying GDP function is Translog, with the important caveat that $p_{A,t}$ must be interpreted as a *geometric* average as given by (11). Moreover, it would merely be a measure of the terms-of-trade effect, rather than of the actual trading gains: the relative-price effect (A9) – which would then be identical to the real exchange-rate effect given by (15) – would still have to be added to obtain the full trading gains, whereas deflating the trade account by $p_{N,t}$ yields the full trading gains directly. While this correspondence is valid for the Törnqvist (and the Cobb-Douglas) aggregation, it does not hold for the Laspeyres aggregation due to the linearity of the corresponding underlying model.