Shadows and donuts: The work-from-home revolution and the performance of cities

Authors:
Steven Bond-Smith[x]
University of Hawai'i
Philip McCann[y]
The University of Manchester

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[x] University of Hawai'i Economic Research Organization
[y] Alliance Manchester Business School and The Productivity Institute

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Authors' contacts:
bondsmit@hawaii.edu; Philip.McCann@manchester.ac.uk

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The Productivity Institute, headquartered at Alliance Manchester Business School, The University of Manchester, Booth Street West, Manchester, M15 6PB. More information can be found on The Productivity Institute's website. Contact us at theproductivityinstitute@manchester.ac.uk

Abstract

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We find that optimal commuting frequency is positively related to the opportunity costs of less-than-continuous face-to-face interaction and inversely related to commuting costs. As well as a "donut effect" with growth in the suburbs and hinterlands around cities, our results also identify a “shadow effect” in smaller cities. The reason is that, somewhat counterintuitively,
commuting frequency optimisation magnifies the benefits of working-from-home in larger cities because of a greater decrease in the burden of commuting. Our results imply enhanced productivity of larger cities over smaller cities, suggesting that the economic divergence between large cities and left-behind places is likely to persist.

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1 Introduction

While the onset of Covid-19 forced many people to WFH, the dominant form of working from home that has since emerged is hybrid (Bloom et al., 2023b) in which many workers still commute to workplaces for face-to-face activities, though not every day. This revolution in work practices has the potential to decentralize economic activity in ways that fundamentally reshape cities. In particular, the technological advances that enable WFH facilitate changes in the frequency with which workers commute for in-person work versus working from home. In turn, workers who commute less frequently can now choose to live further from their workplace or to take jobs in cities further from their home, since they will make the longer commute less frequently (Kim and Long, 2024). Similarly, employers of hybrid workers will be able to recruit from more distant places and access a wider labour market pool. This means that WFH creates a much wider variety of new employment and location choices for both workers and employers with impacts for labour markets and productivity. Many different pieces of evidence posit changes to economies, labour markets, and especially cities, but there is still no consensus on the long-term implications of working from home. Indeed, as the world emerges from the covid economy to something ‘normal’, distinguishing what is temporary and transitory from what is fundamental still remains complex.

Information and communication technologies are already fundamental components of modern economies. As such, the opportunity for the combination of internet technologies and knowledge-based production to generate more inclusive growth by shifting the location of economic activity to more places, has already existed for decades. Yet modern economic growth is characterised by a great divergence in the economic performance of different places, with the major share of growth held by only a handful of large innovative cities (Moretti, 2012). Knowledge-based production in industries such as software, bio-medical research, finance, entertainment, and advanced manufacturing has become increasingly clustered in the most expensive, so-called "superstar cities" (Gyourko et al., 2013), in order to share in the agglomeration benefits of high-skill labour market pooling and shared commercial services. If productivity is still supported by mechanisms that generate agglomeration economies, then in many ways, the work-from-home revolution could reinforce these trends. But if the ability to work from home (WFH) makes all places more equal in terms of their economic opportunities, it has the potential to reverse this divergence trend, by enabling people in other places to access the high-value economic and employment opportunities that are currently so concentrated in city centres alongside business services (Althoff et al., 2022).

In this article, we examine the effects of the work-from-home (WFH) and hybrid work revolution on the spatial structures and performance of cities to understand the implications for the economic geography of cities. In order to do this, we first examine the various pieces of evidence currently available which describe the types of changes that cities are facing in response to the WFH revolution. We then outline the types of conceptual issues which these observations give rise to, and in particular, we emphasise the centrality that the choice of commuting frequency plays in determining how workers, firms, and ultimately cities adjust to the WFH revolution. Existing research does not address this issue. To demonstrate the centrality of this point analytically, we first outline a very simple model which explains how the choice of optimal commuting frequency is related to spatial and behavioural features of urban residents. This then allows us to develop a full urban model which investigates how the variable commuting frequency possibilities afforded by the WFH revolution reshape cities. The changes in the frequency of interactions and the ways in which they endogenously influence all other variables, combine to enhance the productivity advantages of larger cities over smaller cities, an observation which as yet has not been generally understood. The model extends to distinct choices for land use between residential and commercial, allowing activity to endogenously sort into zones with differ between both land use and commuting frequencies. In addition to the widely inferred 'donut' effect, our model uniquely also predicts an urban 'shadow' effect that favours larger cities. We refer to nationwide data from the UK which provides initial evidence supporting all of the insights from our model. While the donut effect intuitively suggests that the WFH revolution will bring about more inclusive growth by increasing welfare in more distant locations, we conclude that the divergence between large cities and left-behind places is likely to persist as the benefits of the WFH revolution are greatest in the regions within commutable distances of larger cities.

2 The work-from-home revolution

Prior to the Covid-19 crisis, for many workers across the globe, working from home was only ever a marginal aspect of their work practices, if at all. Both between firms and within firms and organisations, most important discussions on most topics took place in person. Contract and sales negotiations, personnel training and mentoring activities, as well as hiring, promotion (Emanuel and Harrington, 2021) and human resource activities, were almost all entirely based on face-to-face (FTF) interactions. Indeed, beyond individual firm boundaries, face-to-face interactions are essential also for generating and transmitting the tacit knowledge which is central to driving agglomeration spillovers as well as many aspects of entrepreneurship and innovation-related activities. The result was that prior to the Covid-19 lockdown experience, working from home and working at the workplace (WAW) were only slightly imperfect substitutes for each other (de Graaff and Rietveld, 2007), and although teleworking from home extended overall working hours (de Graaff and Rietveld, 2004) it only ever played a small role in shaping overall working practices (de Graaff and Rietveld, 2007). Prior to the Covid-19 lockdown, less than 5% of UK workers primarily worked from home (Felstead and Feuschke, 2020).

The onset of the Covid-19 pandemic fundamentally changed this situation. Working from home suddenly became a central feature of the working routines of hundreds of millions of workers, with a rapid rise in the use of technologies facilitating online meetings such as Zoom, Microsoft Teams, GoogleMeet, Cisco Webex, Skype-for-Business and others, a phenomenon which has been labelled collectively as the 'zoomshock' (Fraja et al., 2021). The rapid 'zoomshock' learning which societies have undergone in the last few years have changed how both individual workers and organisations consider their employment roles. Widespread evidence (Dingel and Neiman, 2020) suggests that the pandemic-induced WFH tele-working possibilities favoured higher skills and higher income groups, especially in high-value service industries, and those in managerial, professional or financial occupations, relative to all other skills and income groups (Bloom et al., 2023a,b; Sostero et al., 2023). As such, these are the occupational groups who were most able to take advantage of these technologies and to learn how best to work-from-home (WFH), both during the lockdowns but also potentially after the lockdowns have finally ended. The likely persistence of these hybrid working practices (Bloom, 2022) which allows for a greater role of WFH, arises both from the fact that firms and individuals have learned how to better adapt to the new technologies (Bartik et al., 2020), as well as the fact that reduced commuting offers professional and personal benefits (Haldane, 2020; Bruce-Lockhart et al., 2021) including enhanced personal wellbeing (JLL, 2020) and the management of lifestyle choices (Sawhill and Katherine, 2020; Bangham and Gustafsson, 2020). The pandemic experience has helped people to better distinguish between how important and necessary FTF interactions are versus online WFH interactions in different circumstances and with regard to different issues. In particular, people and organisations have learned new modes of working and have discovered that many, but not all, activities, can be done remotely, with no need for face-to-face interaction. On the other hand, evidence has emerged that virtual meetings inhibit innovation compared to FTF interactions (Brucks and Levav, 2022), and the reduced interpersonal communication opportunities associated with WFH also reduces productivity for many higher skilled employees (Gibbs et al., 2022).

As such, across workers there are likely to be a continuum of outcomes. Some workers and firms will conclude that given the nature of their occupations and roles, that their work will continue to be completely in-person and reliant on continuous FTF interactions at the workplace with zero WFH possibilities. In contrast, in other cases, firms and workers will conclude that given a person's occupation and activities, a shift to permanent WFH with zero FTF in-person interactions at the workplace, is sensible for some workers. Finally, some 40% of workers will deploy hybrid working practices which mix FTF with WFH (Bloom et al., 2023b). The overall effects of WFH and hybrid working on the productivity of cities (Behrens et al., 2024) and the wider economy (Mischke et al., 2021) will depend on the balance between the introduction of new information and communications technologies, changes in work practices including working-from-home, and the implications for agglomeration processes.

As yet, however, there are no clear conclusions as to how these changes will play out in terms of economic geography. The different pieces of evidence emerging still leave many questions unanswered. For example, if there is a flattening of the intra-urban land market due to increased WFH practices, what is the effect of this on labour markets, cities, and productivity? One argument is that city centres are especially vulnerable to hybrid working changes (Althoff et al., 2022; Gupta et al., 2022) because their economies are very much driven by face-to-face knowledge spillovers. On the other hand, there may be countervailing processes. For example, it could be that many people become more productive due to adopting either full time WFH or hybrid WFH practices, relative to their former full-time in-person presence in the workplace (Nathan and Overman, 2020). This may partially offset city centre productivity losses. At the same time, city centre firms may also be able to downsize or reconfigure their floorspace and office environments in order make better use of their in-person worker time (Mackenzie, 2021), aimed at maximising knowledge exchanges and spillovers within the firm wherever possible while also cutting out as many office-based routine and non-knowledge-intensive work activities as possible. These changes should enhance firm productivity. On the other hand, excessive WFH may inhibit inter-firm urban agglomeration spillovers (Nathan and Overman, 2020; Behrens et al., 2024) thereby reducing firm productivity. Similarly, if workers require increased compensation for the increased residential floorspace required for WFH and hybrid working, this may also reduce firm profitability (Stanton and Tiwari, 2021). Taking all of these considerations into account, the maximum productivity of the city is likely to be somewhere between full in-person presence in city centres and full WFH patterns (Behrens et al., 2024).

These various insights, however, do not tell us anything about the ways in which the persistent effects of the 'zoomshock' affect how large urban economies perform relative to other types of places. Some commentators argue that WFH provides new growth and economic development opportunities to peripheral regions, but this begs the question as to what exactly we mean by 'peripheral'. Does the ability to WFH nowadays make all places more equal in terms of their development opportunities, thereby potentially narrowing inter-regional inequalities? Alternatively, are the effects of WFH on peripheral areas really only related to places on the urban fringe? More fundamentally, does the WFH revolution actually favour certain types of places over others, thereby potentially widening interregional inequalities? And are there any new types of hinterland effects which may alter the pre-existing urban hierarchy as labour markets expand their geographic reach? These questions still remain largely unanswered, but there is some tentative evidence providing some pointers at the likely implications. UK evidence suggests that large cities could potentially double or even triple their commuting hinterlands if workers cut down their commuting on average from five to three days per week, while spending the same overall time commuting per week (Hellen, 2021). These observations imply that hybrid working may not only lead to intra-urban spread effects, but also that the WFH revolution may also engender new and complex competition effects between cities and regions (Muro and You, 2021), with some smaller towns which already have weaker economies being especially vulnerable (Eley and Hammond, 2020).

In order to understand the types of questions which arise from the WFH revolution, it is useful to consider these issues by referring to a simple diagrammatic framework. In Figure 1 there are two cities: one large city X which is highly productive and has a large hinterland, and one smaller city Y with lower productivity. The convex downward-sloping city bid-rent land-price gradients are given as R_X and R_Y, respectively, and these reflect both the productivity performance of the city and the urban commuting relationships, which are assumed to occur on a daily basis. City X has higher productivity work occuring in its city centre and therefore generates a much larger hinterland than city Y, which only has a small hinterland. We assume that prior to the 'zoomshock', the hinterlands of the two cities did not overlap or encroach on each other and that this system of two cities is in spatial equilibrium such that the marginal resident in each city is indifferent in their location choice both between cities and to other locations within their own cities.

The empirical evidence outlined above suggests that the advent of widespread teleworking and video-conferencing leads to a pivoting and flattening of the bid-rent curve in both cities, for a given level of city-centre productivity. As we see in Figure 2 this results in a 'donut effect' that predominantly increases the hinterlands around both cities. There are greater commuting cost savings in the larger city since workers commuting over longer distances will commute to work less frequently if they can sometimes WFH. The bid-rent curve for the small city will only flatten slightly and remain largely unchanged because commuting in the smaller city is much less burdensome and is likely to continue with little change in frequency. In particular, the greater expansion of the larger city occurs because of greater gains in welfare received by residents located in its hinterland and beyond where commuting cost savings are greatest due to a larger decline in commuting frequency from more distant locations. Starting from an intial spatial equilibrium, these greater commuting cost savings in the hinterlands of the larger city also mean that the marginal resident in City X now achieves a higher utility than the marginal resident in City Y. Returning to spatial equilibrium where marginal residents are indifferent between cities and between locations within cities requires migration from the smaller City Y to the larger City X, as observed in Figure 3.

The larger spatial hinterland for city X increases the job-matching possibilities for hybrid workers in city X, including poaching workers from city Y. These new job-matching mechanisms may also mitigate against falling productivity or land prices in the CBD of city X. Indeed, if WFH allows for better sorting and matching over larger hinterland regions, it may be that city X enjoys both rising productivity in its CBD as well as an expanding hinterland. If these effects are strong enough, they could cast a further economic shadow over city Y. This implies an even greater extension of the distances for an urban shadow effect (Cuberes et al., 2021; Partridge et al., 2009), with gains for places close enough to commute to City X and declines in places just beyond. Moreover, even if city X does experience falling CBD productivity and land prices in the short-term, the flattening bid-rent curves may still allow it to poach hinterland workers from city Y. In contrast, city Y is likely to lose some of its already-smaller CBD-related agglomeration advantages, as more productive workers sort into a higher productivity location in City X, exacerbating its shadow effect, unless it is able to maintain its CBD productivity while also experiencing a flattening of its own bid-rent curve. These examples suggest that in some circumstances, the new hybrid working practices may offer large and prosperous cities even greater advantages over smaller cities, an observation which has not been widely discussed.

Addressing these types of issues has, until now, not been possible, for one key reason; namely, that these effects depend on the explicit choices of workers and firms regarding the frequency of commuting from the home location to the workplace location, and none of the existing handfull of analytical research articles aiming to examine spatial effects of WFH have addressed this issue head on. Yet, it is the commuting frequency which is the key decision-making variable which endogenously reshapes the relationshiops between other spatial and non-spatial variables, and by ignoring this, these papers are only able to indirectly incorporate economic geographny into their models rather than directly and fundamentally. Indeed, empirical evidence supports reduced commuting frequency as a key component of the observed flattening of the bid-rent curve in response to the WFH revolution (Kim and

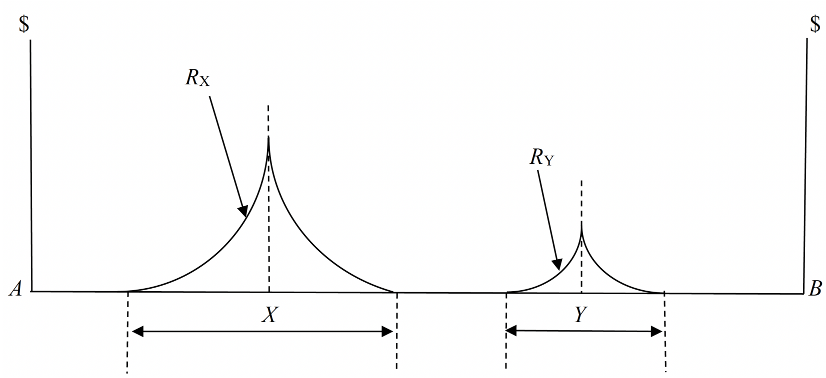

Figure 1: A two-city one-dimensional economy pre WFH

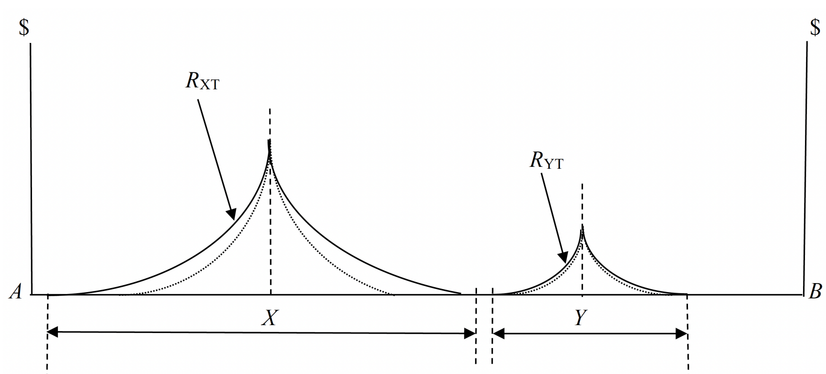

Figure 2: A two-city one-dimensional economy post-WFH with the donut effect

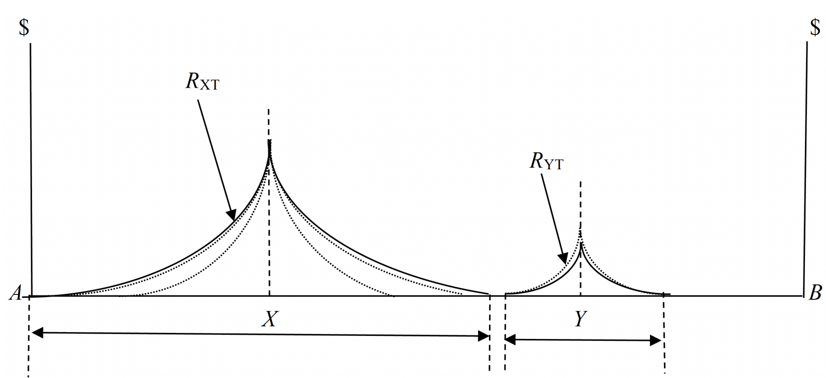

Figure 3: A two-city one-dimensional economy post-WFH with the shadow effect

Long, 2024), but frequency has not been previously applied to examining the implications of WFH in urban economic theory. All of the small number of existing analytical papers (Delventhal et al., 2022; Brueckner et al., 2023; Behrens et al., 2024; Davis et al., 2024) aiming to examine the spatial effects of WFH treat the time in the workplace as simply another parameter and ignore the endogeneity between travel costs, FTF interations and time in the workplace, and the frequency of commuting. In terms of time in the workplace and FTF interactions, Delventhal et al. (2022); Brueckner et al. (2023); Behrens et al. (2024); Davis et al. (2024) treat commuting frequency only indirectly by discussing the share of time allocated to in-person activities at the workplace varying between zero and 1 without addressing the endogenous relationships between travel costs and travel frequencies explicitly, while Liu and Su (2022) do not address this issue even indirectly, instead characterising workers whose share of WFH is either zero or 1. In terms of the endogeneity between travel costs and the commuting frequency, Brueckner et al. (2023) note that WFH or hybrid working reduces the per-mile transport costs t for any distance x between the home and the workplace, although again, they do not address the fact that t is not exogenous and itself depends on the total vehicle miles undertaken, which itself in turn depends on t (Delventhal et al., 2022), something first explained by McCann (1995, 2001). In order to try to partially overcome this problem Delventhal et al. (2022) estimate these total vehicle miles on the basis of empirical data, and then insert these calibrated values into a model structure, rather than developing the location and WFH model itself from the commuting frequency optimisation problem. However the variety of empirical observations means that this indirect empirical approach may not be relevant in different countries and city-region contexts, and this approach still leaves unanswered the core analytical issue which needs to be examined in order to identify these potential heterogeneous hinterland effects of hybrid WFH.

The fact that the endogeneity between the commuting frequency, travel costs, and the intensity of FTF interactions has not been addressed explicitly in any of the current analyses is very important, because the 'zoomshock' has allowed the frequency of commuting to become an explicit and key choice variable in the decision-making processes of both firms and households. The zoomshock-induced hybrid working opportunities involving some WFH practices have inherently forced individual workers and organisations to reconsider the nature of their spatial relationships, in the sense of reconsidering where they live, where they work, and how often they commute so as to interact via in-person face-to-face (FTF) contact, relative to remote interaction via WFH-based telecommuting.

These spatial reconsiderations force people to re-assess the efficiency and effectiveness of commuting behaviour and to adjust their working practices to better fit the commercial, organisational and personal opportunities associated with online working. People's location choices and their time spent working from home will depend explicitly on their choice of commuting frequency, and in turn their chosen commuting frequency will also endogenously determine their total travel time and travel costs, for any given exogenous set of transport prices. Importantly here, there are specific and non-linear relationships between exogenous transport or vehicle costs and total commuting-frequency costs which are endemic to all frequency optimisation problems (McCann, 2001), and which are evident in the geography of knowledge-related transactions (McCann, 2007; Brunow et al., 2020) and in international commuting (McCann et al., 2010; McCann, 2011). In order to be analytically and empirically meaningful the model approach must be consistent with the non-linear relationships in a context where the WFH and hybrid working possibilities mean that the choice of commuting frequency now becomes an explicit decision-making variable for firms and workers. As such, these specific relationships need to be included in any 'zoomshock' related spatial economic model in order to understand the wider economic geography implications of the WFH revolution.

3 A simple example of commuting frequency optimisation

In order to begin to understand why to incorporate travel frequency explicitly into the economic framework, and how we might model these issues, we can consider how both the cost of commuting and the revenue attainable by the firm, both become endogenous to location and commuting frequency decisions. We model the productivity effects of working from home as an opportunity cost because it offers an intuitive comparison to the pecuniary costs of commuting to work. This approach has an advantage over including commuting frequency elsewhere in the firm's cost function since it precisely explains the tradeoffs that commuting frequency imposes on commuting costs and productivity. To explain this importance before we explore a more sophisticated model, this section shows how these fundamental trade offs can be easily and intuitively disentangled by optimizing these tradeoffs between commuting costs and opportunity costs.

Consider a very simple extension of the Alonso-Muth-Mills (AMM) model in which workers can either WFH or undertake face-to-face work in the city centre CBD, denoted Z. $C(d, f)$ is a function of distance and frequency that represents the total pecuniary and opportunity costs associated with less-than-continuous face-to-face contact with clients, customers and suppliers in the CBD, due to WFH. The optimal frequency of commuting balances the trade-offs between saved commuting costs and any reduction in wages due to lower productivity if there is less than continuous face-to-face contact. Adopting the approach of McCann (1995, 2007) in its simplest terms, the worker's total pecuniary and opportunity costs function with respect to commuting frequency to the CBD can be written as:

$$C = \phi d^{\rho} f^{n} + \theta f^{-m} \tag{1}$$

where d is distance, f is commuting frequency, ϕ is distance costs per km, and ρ, n, and m are all positive constants.

In equation 1, the distance costs are a function of the frequency of commuting where ϕd^{ρ} represents the level and structure of the costs with respect to distance for each individual trip.[1] Meanwhile, θ represents the opportunity costs in terms of wages of having less-than-continuous face-to-face contact with customers and suppliers at the city centre CBD, Z. The situation of continuous face-to-face interaction with clients at the city centre CBD is defined as where $f \to \infty$.[2]

However, the opportunity costs associated with the lost revenue due to less-than-continuous face-to-face contact at Z can also be re-written as a mark-up ψ on the rent per square metre paid at the city centre, ψr_z, as this reflects the general agglomeration-related advantages for working in a CBD location. In a competitve labour market, the entire opportunity cost is reflected in the opportunity cost to wages. As such, the total opportunity costs to the worker (including foregone wages) that are associated with less-than-infinite commuting frequency from a residential location to the CBD can be written as:

$$\theta f^{-m} = \psi r_z f^{-m}. \tag{2}$$

We can assume for the moment that the floorspace requirement of the firm is fixed, although shortly we will show how this issue can also be examined. This allows us to re-write equation

1 as:

$$C = \phi d^\rho f^n + \psi r_z f^{-m} \tag{3}$$

Differentiating equation 3 with respect to the commuting frequency f and setting to zero we can derive the optimal commuting frequency of workers from any given location at a distance d away from the CBD. Thus.

$$\frac{\partial C}{\partial f} = n\phi d^\rho f^{n-1} - m\psi r_z f^{-m-1} = 0 \tag{4}$$

which gives:

$$\frac{m\psi r_z}{n\phi d^\rho} = f^{m+n}. \tag{5}$$

For a worker located at a distance d from the CBD, the optimal commuting frequency f^* from d to Z is given by:

$$\left(\frac{m\psi r_z}{n\phi d^\rho}\right)^{\frac{1}{m+n}} = f^*. \tag{6}$$

Equation 6 tells us that for any given worker at a location distance of d away from the CBD, Z, the optimal frequency f^* of commuting of the worker to the CBD is positively related to the opportunity costs of less-than-continuous face-to-face interaction and inversely related to the travel plus travel-time costs. In the normal case where $n = m = \rho = 1$, then the optimal commuting frequency f^* is a square root function of each of the various parameters. Longer commuting distances d have lower optimal commuting frequencies while activities with higher agglomeration-related advantages, generating a higher city centre rent premium, will tend to exhibit higher commuting frequencies. While the example above is simplified to involve only workers, the same principles also apply to firms. As shown in the complete model below, the city-wide urban bid-rent curves are also constructed on the basis of endogenously determined optimal commuting frequencies f^* which are uniquely determined for each respective distance d.

4 A model of the city with commuting frequency

Based on this foundation, we expand the AMM city structure to include both workplaces and homes and characterise bid-rent curves, density, spatial distribution and optimal commuting behaviour as workers and firms respond to the WFH revolution. The inclusion of workplaces in the standard urban model allows firms to make differing location and sorting decisions alongside workers while also negotiating agreements with workers regarding commuting frequency. The usual AMM approach, in which all commercial acitivty is assumed to occur at the city centre, results in a corner preference for for firms such that the entire commuting travel cost and WFH opportunity cost falls on workers. Our approach also allows for firms to participate in the commuting frequency and relocation trade-off, relocating further from the city centre in order to accommodate a workforce that commutes less frequently. This also allows an elegant solution to the current debate between return to office mandates and continued hybrid WFH that will still take some time to reach an equilibrium agreement between firms and workers. As is shown below, it is an intuitive solution to resolve how the costs and benefits of WFH are shared between workers and their employers by allowing both firms and workers to relocate, and adjustments in wages, in order to agree on commuting frequencies. Furthermore, it offers insights beyond the homogenous firm-worker model here in terms of heterogeneous industries where the need and ability for workers to WFH differs. These logical intuitive extensions allow us to consider how the WFH revolution is likely to affect workers and firms in different types of industries, labour markets, and cities, providing a rich and descriptive insight into the performance of cities from an otherwise standard homogenous urban framework.

As above, the city is centred at Z. The main differences between this model and other AMM-type models is that this city has two types of buildings, housing and offices. As in the simple example above, we optimise commuting behaviour. This allows for a more realistic city structure and sharing of commuting costs between firms and workers. Consumers have a taste for larger homes, while demand for office space is inelastic. As a result, offices cluster around near the city centre and homes sit in the region beyond the commercial area. Workers produce goods with constant returns to scale which can be transported costlessly around the city.

4.1 People

Workers inelastically supply their labour in the city centre and—for WAW—commute from their homes to the city centre. We extend the AMM framework to accomodate office space for firms, which hire workers in the city centre and pay for them to commute from the city centre to their workplace. These assumptions about commuting route via the city centre are a natural extension of the AMM city structure for analytical convenience that accomodates two building uses and generates an intuitive analytical description of the allocation of commuting costs between workers and firms. In reality, this cost may be less and its allocation will be subject to a negotiation between firms and workers, though a competitive outcome would by similar since it would imply that the costs fall where they are incurred—that is, commuting cost-sharing reflects the relative market power of workers and employers based on their respective locations.

People living in the city choose their home location d_h relative to the city centre Z where $d = 0$, the size of their home h and consumption of final goods y to maximise utility,

$$\max U(d_h, y, h) = y^{1-\mu} h^{\mu} \qquad 0 < \mu < 1, d_h \geq 0. \tag{7}$$

People face a budget constraint that their income is sufficient to pay for commuting costs to the city centre, consumption of final goods and consumption of home space

$$w(Z) \geq \phi d_h f^n + \theta_h f^{-m} + y + h.r(d_h) \qquad d_h \geq 0 \tag{8}$$

where f is the frequency of commuting, ϕ is a distance-based commuting cost, $w(Z)$ is the market wage in the city centre Z, θ_h is reduced wages that the worker might have to incur due to a fall in productivity when working from home and $r(d_h)$ is the rent per unit of home space in the location of their home. Other centripetal forces could also operate but for simplicity we assume commuting to the city centre is the only centripetal force in the model for residents. Commuting frequency can range between zero and an unspecified maximum, denoted f_{max}, which can be thought of as commuting every (work) day so that no work occurs at home, as is explicitly assumed in standard urban economic models (Fujita, 1989). In spatial equilibrium people are indifferent between locations, $\frac{\partial U}{\partial d_h} = 0$. Rent and commuting costs are paid to local landlords and transport providers who consume only final goods. This allows general equilibrium effects from changes in land value without introducing a cumbersome redistribution externality into location decisions.

Standard optimization of Cobb-Douglas utility implies that consumers spend a constant portion of their income after commuting costs on housing space and the remaining share on final goods.

$$\mu\left(w\left(Z\right)-\phi d_h f^n-\theta_h f^{-m}\right)=h.r\left(d_h\right) \tag{9}$$

and

$$\left(1-\mu\right)\left(w\left(Z\right)-\phi d_h f^n-\theta_h f^{-m}\right)=y. \tag{10}$$

For a homogeneous population, all residents have the same utility. Totally differentiating $U\left(r\left(d_h\right),w\left(Z\right)-\phi d_h f^m\right)$ with respect to distance, setting to zero since utility is equal in spatial equilibrium, and rearranging, the slope of the bid-rent curve in the residential area is:

$$\frac{\partial r\left(d_h\right)}{\partial d_h}=-\frac{1}{\mu}\frac{r\left(d_h\right).\phi f^n}{w\left(Z\right)-\phi d_h f^n-\theta_h f^{-m}}=-\frac{\phi f^n}{h\left(d_h\right)} \tag{11}$$

where $h\left(d_h\right)$ is the solution found above in equation 9. This meets the Alonso-Muth condition that $\frac{\partial r(d_h)}{\partial d_h}<0$ for the region of the city containing homes.

4.2 Firms

Firms choose their location relative to the city centre to maximise profit subject to the free entry condition that revenue is just sufficient to pay for office space, wages and the employer's share of commuting costs required to entice employees to commute from the city centre to the workplace for work at work, $\pi_i=0$. The maximisation problem for a firm located at d_o is:

$$\max\pi=\left(p_i-\left(w\left(Z\right)-\theta_h f^{-m}+\phi.d_o.f^n+\overline{a}.\theta_o f^{-m}+r\left(d_o\right).g\right)\frac{1}{\overline{a}}\right)x_i \tag{12}$$

subject to the free entry condition $\pi_i=0$ where $\overline{a}$ represents the symmetric productivity of homogeneous workers conducting their work at work, x_i is output of final goods by firm i, θ_o is the opportunity cost in terms of reduced output due to changes in the productivity when workers WFH instead of WAW, θ_h is the reduction in wage costs due to lower wages for workers who WFH to reflect this reduced productivity, and g is the units of office space required per employee. Office space g is initially treated as a parameter, but in Section 5.2 we consider it as a function of commuting frequency in which workers require less office space if they commute less frequently.[3]

Worker productivity is a function of the frequency that they commute to WAW versus WFH. We could simplify the firm's problem by formulating $\overline{a}$ as a function of commuting frequency, but the opportunity cost specification described above offers an intuitive optimisation of the trade off between the costs and benefits of WAW versus WFH. Our assumptions about θ_o and θ_h reflect a simplification of the equilibrium between commuting cost savings and diminishing marginal productivity at home and work. Specifically, the foundation for these parameters implicitly assume that in equilibrium a worker's *marginal* productivity is lower at home than at work—which is confirmed by market preferences for hybrid WFH rather than full WFH—without modelling the exact nature of marginal productivity. We cannot be sure yet if workers are less productive at home overall, as some have reported increased productivity for hybrid WFH (Bloom et al., 2023b) and others decreased productivity—it is simply too early to draw conclusions on this point—so we make no assumptions about potential changes in $\overline{a}$ nor is that the focus of this article. Productivity may well be higher at home for many tasks, but there will be greater diminishing marginal productivity for WFH compared to WAW because some tasks can only be performed in person. An equilibrium with a hybrid combination of WFH and WAW reveals that the marginal productivity of WFH is lower than WAW.[4] Indeed, recent research reveals that the dominant form of remote work is hybrid (Bloom et al., 2023b) and that full WFH is associated with lower productivity (Barrero et al., 2023).

Since there is free entry, firms can choose to locate anywhere and there is costless trade within the city. In spatial equilibrium rents at each location adjust such that firms have the same marginal cost of production which allows for symmetric firms. i.e. $c = (w(Z) + \phi.d_o.f^n + (\overline{a}\theta_o - \theta_h) f^{-m} + r(d_o).g) \frac{1}{\overline{a}}$ is the same for all firms and locations where firms operate with frequency of commuting and rent adjusting for different firm locations. Free entry also means that firms are small enough that they ignore the effect of their own demand for office space and workers on the price of office space or the wage rate because firms and workers are mobile, firms are indifferent between locations and any individual firm is too small to affect city-wide density or wages (i.e. $\frac{\partial r(d_o)}{\partial p_i} = 0$ and $\frac{\partial w(d_o)}{\partial p_i} = 0$). These assumptions are plausible with a sufficiently large number of firms.[5]

Constant returns to scale means that price equals marginal cost:[6]

$$p_i = \left(w(Z) + \phi.d_o.f^n + (\overline{a}\theta_o - \theta_h) f^{-m} + r(d_o).g\right) \frac{1}{\overline{a}}. \tag{13}$$

In spatial equilibrium, with costless trade, all firms have the same prices. Totally differentiating price with respect to distance d_o, setting to zero since prices of final goods are the same everywhere in spatial equilibrium, and rearranging finds the slope of the bid-rent curve:

$$\frac{\partial r(d_o)}{\partial d_o} = -\frac{\phi f^n}{g}. \tag{14}$$

That is, for each additional unit of distance rent decreases according to the change in frequency of commuting and the office space required per employee. This differential equation also meets the Alonso-Muth condition that $\frac{\partial r(d_o)}{\partial d_o} < 0$ for the region of the city containing office space.

Finally, the two building uses, office and home space, sort into different districts of the city. The market-based spatial sorting of offices and homes means that the building type with the steepest bid rent curve is closer to the city centre because bidders with a flatter curve could outbid their opponent simply by moving further from the city centre. Examining equations 11 and 14, offices will be located in the city centre so long as the homes that would be built in that location are larger than the offices,

$$h(d_h) > g. \tag{15}$$

4.3 Optimal commuting frequency

Commuting costs from homes to offices via the CBD are already apportioned above on the basis of location choices: Workers pay the cost of commuting from their homes to the CBD and firms pay the cost of workers commuting from the CBD to the office. When workers commute less frequently, the commuting cost savings initially accrue to workers, but firms will reduce wages due to lower productivity, allowing firms to also receive some of the commuting cost savings. Workers will only be willing to accept lower wages if the savings from reduced commuting are still sufficient to make workers better off than their wage at a higher productivity level with daily commuting. In equilibrium, there is an optimal agreed frequency of commuting and reduction in wages that also shares the cost of reduced productivity due to hybrid WFH.

Substituting equilibrium consumption of final goods and home space into the utility function (Equation 7) and differentiating with respect to the frequency of commuting, setting to zero, and rearranging gives optimal commuting preferences for workers. For a worker living at a distance d_h from the city centre, the optimal commuting frequency as a function of their reduced wages and location relative to the city centre is:

$$f_h^* = \left(\frac{m\theta_h}{n\phi d_h}\right)^{\frac{1}{m+n}}. \tag{16}$$

Similarly, differentiating firm profit (Equation 12) with respect to frequency of commuting and rearranging gives optimal commuting frequency for firms. For a firm with an office located at distance d_o from the city centre the optimal commuting frequency as a function of reduced productivity and the location of the office relative to the city centre is:

$$f_o^* = \left(\frac{m\left(\bar{a}\theta_o - \theta_h\right)}{n\phi d_o}\right)^{\frac{1}{m+n}}. \tag{17}$$

Note that even in this more complex model with a two-activity land-use pattern, these results are essentially the same as in the simple model in Section 3 (Equation 6).

In order to agree on commuting frequency, firms and workers will negotiate the reduction in wages that capture the change in each worker's productivity untill a labour market equilibrium is reached where each firm-worker location pair has an individual optimal commuting frequency. Combining these commuting frequency preferences for workers and firms and rearranging finds that the required reduction in wages is:

$$\frac{d_h}{d_o + d_h}\bar{a}\theta_o = \theta_h. \tag{18}$$

in order to agree on commuting frequency in equilibrium. This means that workers bear a portion of the opportunity cost to productivity based on their share of commuting costs. Workers also save their $\frac{d_h}{d_o+d_h}$ share of overall commuting costs. Similarly, firms bear a portion of the opportunity cost to productivity equal to the firm's $\frac{d_o}{d_o+d_h}$ share of commuting costs and save a $\frac{d_o}{d_o+d_h}$ share of overall commuting costs. Taken together, workers will undertake hybrid-WFH if the overall reduction in productivity per day due to working from home is less than the overall cost of commuting, generating a productivity improvement overall. By substitution the equilibrium commuting frequency for each firm-worker pair is

$$f^* = \left(\frac{m\bar{a}\theta_o}{\left(d_o + d_h\right)n\phi}\right)^{\frac{1}{m+n}}. \tag{19}$$

As we noted above, the opportunity costs associated with the lost revenue due to less-than-continuous face-to-face contact can also be re-written as a mark-up ψ on the rent per square metre paid at the city centre $\theta_o = \psi r\left(0\right)$, as this reflects the general agglomeration-related advantages of a more central location. By substitution, optimal frequency is

$$f^* = \left(\frac{m\bar{a}\psi r\left(0\right)}{\left(d_o + d_h\right)n\phi}\right)^{\frac{1}{m+n}}. \tag{20}$$

This allocation of commuting cost savings and opportunity costs between workers and firms, and the resulting optimal frequency of commuting, are efficient since costs are apportioned on the basis of each party's location decision and frequency is determined by joint profit and utility maximisation. Workers face a distance-based share of commuting costs based on the location of their home and a distance-based cost of lower productivity per day working from home. This is efficient since their preference for frequency of commuting is a function of their distance from the CBD. Similarly, firms face a cost of commuting and an opportunity cost of lower productivity per day when workers are working from home, based on the location of the office relative to the CBD. While these analytical results are based on our simplifying assumption that commuting occurs via the CBD, it implies that the pecuniary and opportunity costs and benefits of working from home will be apportioned in the same way that commuting costs are already apportioned, which may be less than we assume.

Of course, while this apportioning is efficient, in reality, the actual commuting frequency outcomes for individual workers in individual firms will depend on both firm and worker preferences (Aksoy et al., 2022), as well as bargaining practices and legal powers which differ across countries, states and sectors. That said, if labour market conditions varied from our commuting cost apportionment, $\frac{d_h}{d_o+d_h}$, the only changes to these equations are to this apportionment. If labour market conditions favored firms, this ratio would be larger, meaning that commuting costs are largely paid by workers. As a result, our model implies that workers would pay for all of the opportunity cost of reduced productivity and would even pass on some of their commuting cost savings to firms by accepting a wage reduction that is greater than this opportunity cost. Both firms and workers would relocate in order to reach a spatial and labour market equilibrium. Similarly, if labour market conditions favored workers, then this ratio would be smaller, meaning that commuting costs are mostly paid by firms. As a result, our model implies that firms would largely bear the opportunity cost of reduced productivity and would pass on some of their commuting cost savings through wage reductions that are less than this opportunity cost, if wages are reduced at all. Again, firms and workers would relocate in order to reach a spatial and labour market equilibrium. Thus our model is capable of accommodating a variety of labour market conditions to assess the likely implications for different places.

4.4 Construction

Perfectly competitive developers create building space throughout the city using land (L) and capital (K) with constant returns to scale to produce $B(d)$ units of building space per unit of land at a distance d from the city centre. To simplify the building sector, we assume that construction does not involve labour, meaning that we do not have to consider the commuting behavior of construction workers. In any case, no construction occurs in spatial equilibrium since we assume that population is constant and buildings last indefinitely. The rental price of land is denoted $R(d)$ (as opposed to the lower case for renting building space). Building space can be used for either offices or homes and there is no cost difference in their construction. A property developer faces the maximisation problem:

$$\max \pi = r.B - K - R.L \tag{21}$$

where B is the CRS production function for units of building space, R is the price per unit of land and K is capital. Note that construction occurs without labour and L refers to units of land, not labour. Since the rental price of capital is the same everywhere and exogenous, it is omitted. A developer's profit is zero with perfect competition. The zero profit condition for developers can also be written

$$r(d) = \frac{R(d) + K(d)}{B(d)}. \tag{22}$$

Totally differentiating with respect to distance, rearranging and simplifying on the basis of profit maximisation in Equation 21, the slope of the bid rent curve for land is

$$\frac{\partial R(d)}{\partial d} = \frac{\partial r(d)}{\partial d} B(d). \tag{23}$$

We assume the building space production function takes the Cobb-Douglas form $B = \alpha L^{\beta} K^{1-\beta}$ where α describes the productivity of the construction industry, L is units of land and K is units of capital. By substitution, the maximization problem for construction is:

$$\max \pi_d = r.\alpha.L^{\beta} K^{1-\beta} - K - R.L = 0. \tag{24}$$

Optimisation finds that the land price per unit is a function of the ratio of capital and land.

$$R = \frac{\beta}{1-\beta}.\frac{K}{L} \tag{25}$$

Rearranging the building space production function, the capital to land ratio for a developer is:

$$\frac{K}{L} = \left(\frac{B}{\alpha.L}\right)^{1/(1-\beta)} \tag{26}$$

By substitution:

$$R = \frac{\beta}{1-\beta}.\left(\frac{B}{\alpha.L}\right)^{1/(1-\beta)} \tag{27}$$

By substitution and rearranging, the optimal price of land as a function of building space rent in that location is:

$$R(r(d)) = \beta (1-\beta)^{(1-\beta)/\beta}.(\alpha.r(d))^{1/\beta}. \tag{28}$$

Alternatively, the rental price of building space can be written as a function of the rental price of land:

$$r(R(d)) = \left(\frac{1}{1-\beta}\right)^{1-\beta} \beta^{\beta}.R(d)^{\beta}.\frac{1}{\alpha}. \tag{29}$$

4.5 Closing the model

We close the model by exogenously defining the population of the city as a fixed parameter N, and solving the equilibrium by distributing population and jobs such that utility is maximised and equalised across locations, since all residents and jobs are otherwise homogeneous. The resulting conditions define the extent of the city and the regions containing offices and homes. Since space required for jobs is inelastic, but consumers have a preference for home space, it follows that the central city will be used for office space, and the surrounding region used for homes. Starting in the city centre, building space will be used for office space if its value exceeds the value of using it for homes. Developers build if the value of land containing buildings of any kind exceeds the value of land not used for buildings (i.e. some other alternative use such as agriculture $R(A)$).[7]

The region of the city containing offices must be sufficient to employ the population and the region containing homes should be sufficient to hold homes for the population $N = \int_{-\tilde{d}_o}^{\tilde{d}_o} j(d)\, dd$ and $N = \int_{-\tilde{d}_h}^{-\tilde{d}_o} n(d)\, dd + \int_{\tilde{d}_o}^{\tilde{d}_h} n(d)\, dd$ where $j(d)$ is the density of jobs to land and $n(d)$ is population density. In a symmetric city, this can be written:

$$N/2 = \int_0^{\tilde{d}_o} j(d)\, dd \tag{30}$$

and

$$N/2 = \int_{\tilde{d}_o}^{\tilde{d}_h} n(d)\, dd. \tag{31}$$

Using equations 14 and 23 the density of jobs hosted to land is

$$\frac{B(d)}{g} = j(d_o) = -\frac{1}{\phi f^n} \frac{\partial R(d_o)}{\partial d_o}. \tag{32}$$

in the region containing offices. Similarly using equations 11 and 23 population density is

$$\frac{B(d)}{h(d_h)} = n(d_h) = -\frac{1}{\phi f^n} \frac{\partial R(d_h)}{\partial d_h}. \tag{33}$$

in the region containing homes. Substituting into the market clearing conditions above (equations 30 and 31), solving the integrals, and rearranging finds that land rent in the city centre is

$$R(0) = R(A) + \phi f^n N. \tag{34}$$

In spatial and labour market equilibrium optimal commuting frequency means that workers will sort into into home locations based on unique office and home location pairs where each worker and firm can agree on a commmuting frequency—those firms in the city centre at $d = 0$ will agree to much more frequent commuting for their employees than firms further from the centre. In order to accomodate optimal frequency commuting preferences, employees of firms located in the city centre will live in homes just beyond the boundary of the commercial district $\tilde{d}_o$. Similarly, those firms at the outer boundary of the commercial district will require less freqeuent commuting, allowing their employees to sort into homes in more distant locations at the city boundary $\tilde{d}_h$. This spatial sorting pattern is efficient, since it will minimize the total amount of commuting. Thus building space rent in the city centre is:

$$r(0) = \left(\frac{1}{1-\beta}\right)^{1-\beta} \beta^\beta . \left(R(A) + \phi (f^*)^n N\right)^\beta . \frac{1}{\alpha}. \tag{35}$$

where f^* is the optimal frequency of commuting to offices given in Equation 19 and evaluated at the city centre with employees located at the residential district at the boundary of the commerical district in order to minimize a reduction in wages: $f^*(d_o = 0) = \left(\frac{m\bar{a}\theta_o}{\tilde{d}_o n\phi}\right)^{\frac{1}{m+n}}$. By substitution, the bid rent curve for building space in the region containing offices is:

$$r(d_o) = \left(\frac{1}{1-\beta}\right)^{1-\beta} \beta^\beta . \left(R(A) + \phi \left(\frac{m\bar{a}\theta_o}{\tilde{d}_o n\phi}\right)^{\frac{1}{m+n}} N\right)^\beta . \frac{1}{\alpha} - \frac{\phi (f^*(d_o))^n d_o}{g}. \tag{36}$$

where $f^*(d_o) = \left(\frac{m\bar{a}\theta_o}{(d_o+d_h^*)n\phi}\right)^{\frac{1}{m+n}}$ in which d_h^* is the distance from the city centre that employees of a firm located at d_o choose to live. Similarly, the bid rent curve for building space in the region containing homes is:

$$r(d_h) = \left(\frac{1}{1-\beta}\right)^{1-\beta} \beta^{\beta}.\left(R(A) + \phi(f^*)^n N\right)^{\beta}.\frac{1}{\alpha} - \frac{\phi(f^*(d_o))^n \tilde{d}_o}{g} - \frac{\phi(f^*(d_h))^n d_h}{h(d_h)} \quad (37)$$

where $f^*(d_h) = \left(\frac{m\bar{a}\theta_o}{(d_o^*+d_h)n\phi}\right)^{\frac{1}{m+n}}$ in which d_o^* is the distance from the city centre where a worker living at d_h would be employed. The boundaries of the commercial district and the residential area can be calculated accordingly where $R\left(\tilde{d}_o\right) = R(d_h)$ and $R\left(\tilde{d}_h\right) = R(A)$ respectively. Substituting optimal commuting frequency into Equation 35 also gives an implicit definition for city centre rent:

$$r(0) = \left(\frac{1}{1-\beta}\right)^{1-\beta} \beta^{\beta}.\left(R(A) + \phi\left(\frac{m\bar{a}\psi r(0)}{\tilde{d}_h n\phi}\right)^{\frac{n}{m+n}} N\right)^{\beta}.\frac{1}{\alpha}. \quad (38)$$

Overall, the model implies that optimal commuting frequency, due to regularly working-from-home, means a lower rent premium in a city centre location, and flattens slope of the bid-rent curve wherever $f^* < f_{max}$ such that rents are higher at more distant locations.

5 How WFH reshapes cities

To understand how the WFH revolution reshapes cities and how it affects different types of cities, we compare outcomes in spatial equilibrium when workers are assumed to commute every day, with outcomes after the WFH revolution, when workers can optimize commuting frequency. The change to an optimal commuting frequency incentivises relocation decisions to return the model to a spatial equilibrium. The extent that outcomes differ across the closed city model specified above describes relocation forces within the city. While the specified model is a closed city, the extent that outcomes differ in different types of closed cities, imply relocation forces *between* cities. This comparison approach allows us to consider the impacts in different types of cities and different systems of cities, while avoiding the additional complication of simulating each arbitrary system of open cities in general equilibirum.

Prior to the WFH revolution, we assume that all workers would be commuting every day, denoted f_{max}, as is explicitly assumed in standard urban economic models (Fujita, 1989). Given this restriction, the population distribution was in a spatial equilibrium in which each worker was equally well off, regardless of their location within a city or in alternative cities of different sizes. The results of the model are unchanged from the above, except that the frequency of commuting is arbitrarily fixed at f_{max} and thus prevented from ever reaching f^* which is less than f_{max} for many workers, as was the case with many office and workplace managerial cultures prior to the Covid-19 lockdowns.[8]

The starting points for analysis are utility given in equation 7 with equilibrium consumption given in equations 9 and 10 and the marginal cost of production which is equal to the competitive price of final goods as shown in equation 13. The WFH revolution reduces the frequency of commuting from f_{max}, in which workers commute every day, to $f^* \leq f_{max}$, in which workers prefer to regularly WFH, shown in equation 19. Initially, we hold prices, rents, and locations as fixed because we need to understand the changes in utility and marginal cost prior to any relocation decisions in order to understand the incentive for workers and firms to relocate to a different part of the city or to another city.[9]

5.1 Relocation within cities and the donut effect

The first and most well known effect of the WFH revolution is a "donut effect" (Ramani and Bloom, 2021), which recognizes that the commuting cost savings of WFH are greater for those who live further from the city centre, incentivizing changes in rents and location as the city returns to a spatial equilibrium. The donut label refers to the shape of these changes: declines in city centres and gains in a donut shape around the city centre.

Prior to the WFH revolution, when the frequency of commuting is f_{max}, the term $\theta_h f_{max}^{-m}$ is essentially zero, since workers are commuting every day and do not WFH. So, we can characterize initial utility as

$$\overline{U}_{WAW} \approx \lim_{f \to f_{max}} U = (1-\mu)^{1-\mu}\,\mu^{\mu}\frac{w(Z) - \phi d_h f_{max}^{n}}{r(d_h)^{\mu}} \tag{39}$$

where this utility is the same for any location in any type of city because the system is initially in a spatial equilibrium in which rents and development are such that all workers are housed and employed and no worker can be made better off by relocating. Utility after the WFH revolution is

$$U_{WFH} = (1-\mu)^{1-\mu}\,\mu^{\mu}\frac{w(Z) - \phi d_h {f^*}^{n} - \theta_h {f^*}^{-m}}{r(d_h)^{\mu}}. \tag{40}$$

The WFH revolution introduces a trade off between commuting frequency and income in which utility increases by reducing commuting costs but decreases due to lower wages (due to lower productivity) when working from home. Prior to any changes in rent or location, the change in utility is $\triangle U = U_{WFH} - \overline{U}_{WAW}$,

$$\triangle U = (1-\mu)^{1-\mu}\,\mu^{\mu}\frac{\left(\phi d_h \left(f_{max}^{n} - {f^*}^{n}\right) - \theta_h {f^*}^{-m}\right)}{r(d_h)^{\mu}}. \tag{41}$$

The net change in utility is zero or positive, by definition, because if there were a decrease in welfare, then a worker could simply increase welfare by incrementally increasing frequency untill they reach f_{max} where there is no decrease. In other words, the welfare of all workers is equal to or improves due to the WFH revolution because welfare increases for workers living in locations where they choose to commute less frequently and sometimes WFH but workers still have the choice to commute every day.

Setting the change in utility to be greater than or equal to zero, substituting the opportunity cost to wages found in equation 18, and solving, implicitly defines the critical trade-off between income and commuting cost savings that determines whether workers choose to sometimes WFH:

$$\theta_h {f^*}^{-m} < \phi d_h \left(f_{max}^{n} - {f^*}^{n}\right) \tag{42}$$

where f^* is the optimal commuting frequency at d_h as shown in equation 16, which itself is a function of d_h. Workers living in a location that satisfies the inequality in equation 42 will choose to sometimes WFH, because the lost wages due to working from home, $\theta_h {f^*}^{-m}$, are less than the savings in the cost of commuting at an optimal frequency, $\phi d_h \left(f_{max}^n - f^{*^n}\right)$, such that $f^* < f_{max}$. Put simply, welfare increases whereever workers would prefer to sometimes WFH and commute less frequently than every day. Rearranging equation 16 with the inequality $f^* < f_{max}$, where the inequality in equation 42 holds, defines this region of the city explicitly in which welfare increases for workers living further away from the city centre than the distance threshold of:

$$d_h > \frac{m\theta_h}{n\phi f_{max}^{m+n}}. \tag{43}$$

For locations at and inside this distance, workers are initially no worse off and will continue to commute every day because the commuting cost saving is less than their opportunity cost of working from home. For locations beyond this limit, workers are unambiguously better off because they can now commute at $f^* < f_{max}$ and the opportunity cost of working from home is less than the commuting cost savings of reducing their commuting frequency.

A similar approach can be applied to examine how the WFH revolution affects the productivity of firms. As with workers, profit (and effective productivity) increases for firms located whereever firms would prefer workers to commute less frequently than every day, such that $f^* < f_{max}$. Rearranging equation 17 with the inequality defines the the region of the city where profit increases for firms located further away from the city centre than the threshold distance of

$$d_o > \frac{m\left(\overline{a}\theta_o - \theta_h\right)}{n\phi f_{max}^{m+n}}. \tag{44}$$

For firms located inside this distance limit, firms would initially insist on workers commuting every day because the firm's share of commuting costs is less than the opportunity cost to productivity for their workers. For firms beyond this distance, profits increase because they can negotiate with workers to avoid the firm's share of commuting costs and these firms are willing to allow workers to sometimes WFH because that ommuging cost saving is greater than lost output when workers sometimes WFH.

As noted, to reach a spatial equilibrium, each firm and worker pair still has to agree on a commuting frequency, given their respective (re)location decisions and preferences. In order to reach an equilibrium where all workers and firms agree on a commuting frequency, workers and firms negotiate a reduction in wages by $\frac{d_h}{d_o+d_h}\overline{a}\theta_o$, as found in equation 18. Substituting and rearranging, the total commuting distance threshold where both firms and workers are unambiguously better off can be described by the inequality

$$d_o + d_h > \frac{m\overline{a}\theta_o}{n\phi f_{max}^{m+n}}, \tag{45}$$

which is also conditional on the individual firm and home location inequalities above. As above, this total commuting distance threshold can also be found by setting the equilibrium commuting frequency shown in equation 19 to be greater than f_{max} and solving for total commuting distance.

Since there is an increase in welfare and profit for workers and firms located in the regions of the city beyond these distances, but no change in welfare in locations less than these threshold distances, this gives an incentive to relocate *within* the city. Demand for office and residential building space in the region inside these thresholds will decline and demand for office and residential building space outside these thresholds will increase. The change in demand incentivises a decline in rent inside these thresholds and an increase in rent beyond them. This phenomenon has become known as the “donut effect” (Ramani and Bloom, 2021) because it pulls workers and firms to relocate further from the city centre in a donut shape around the city.

While other models also show a donut effect, commuting frequency optimization provides a richness that is not seen in any other model. By optimizing commuting frequency, which allows firms and workers to optimize the trade–off between commuting and opportunity costs, we can see that the donut effect is enhanced by the length of a commute, meaning there is a stronger donut effect in larger cities. Commuting from the fringes of a large city is not so burdensome if you don’t have to do it every day. With reduced commuting frequency, foregone commuting increases as a resident chooses to live further from the workplace, but with frequency optimization the reduction in frequency is even greater for longer commuting distances, further enhancing the benefit of the WFH revolution and amplifying the donut effect in larger cities. And by including building space for workplaces in the model and negotiating the allocation of commuting costs between firms and workers, characteristics of the donut effect can be observed in the model in both commercial and home locations. And even with a homogeneous specification for firms and workers, frequency optimization allows for different zones within commercial and residential districts, in which some workers commute every day and others often WFH. This richness was not possible in models that simply parametrise WFH and commuting behaviors.

5.2 The performance of different types of cities

Cities of different sizes perform very differently after the WFH revolution. Equation 41 shows how the change in utility (and also profit) is a function of location relative to the city centre. But cities with larger populations will stretch across greater distances in order to house *and employ* all residents and experience an amplified donut effect. So, larger cities will host a greater share of residents who WFH. The donut effect is amplified in larger cities because those who sometimes WFH in larger cities will also commute less frequently than hybrid workers in smaller cities. The smallest cities may not have any workers at all commuting from such distant locations, so there would be no benefit at all to its workers sometimes working from home—commuting is always less burdensome than the oppoprtunity cost to productivity in small towns.

In the smallest cities, the opportunity cost to productivity of working from home is greater than the relatively small cost of commuting such that $f^* = f_{max}$ for all workers with homes in the smaller city, due to sufficiently low rents in proximate locations to workplaces in those smaller cities. Similarly, the city may be small enough that all firms are close enough to the city centre that the opportunity cost of WFH is always greater than the firm’s cost savings in wages that reimburse workers for the firm’s share of commuting costs. So, in these smallest cities, the model still predicts *hypothetical* welfare gains beyond the estimated distance threshold, even if no workers would have been commuting from those places prior to the WFH revolution.

Yet in the smallest cities and towns, those hypothetical welfare gains may not be sufficient to attract workers to relocate beyond that threshold distance where those workers would choose to sometimes WFH. So, nothing really changes at all in the smallest cities because commuting is not so burdensome that WFH offers those workers any real benefit at all.

In larger cities, the WFH revolution increases utility for workers living anywhere beyond a distance of $\frac{m\theta_h}{n\phi f_{max}^{m+n}}$ and profit increases for firms located beyond a distance of $\frac{m(\overline{a}\theta_o-\theta_h)}{n\phi f_{max}^{m+n}}$. If the threshold for residential location is greater than the treshold for commercial building, then the inner portion of the residential district, and the central portion of the commercial district, will involve workers who commute every day for work at work. In this region of the city there is no change to the *slope* of the bid-rent curves at all because $f^* = f_{max}$, but there is a change in its level. Workers living beyond the threshold will commute regularly, and sometimes WFH. This region of the city has a flattening of the bid-rent curve based on equations 11 and 14 because $f^* < f_{max}$. This flattening of the curve beyond this distance threshold results in a decline in the level of the bid rent curve. The greatest decline occurs in the central region, where workers still commute every day but the decline in this region is even. Beyond the threshold there is also a decline, but the magnitude of the decline gradually diminishes with distance as the slope of the curve is now flatter.

Examining the distance threshold inequality for this mixed WAW-WFH city, substituting the equilibrium wage decline given in equation 16, and rearranging, gives the condition

$$d_o + d_h < 2d_h, \tag{46}$$

which (for now) is always true, since d_o is less than d_h. This rules out the possibility of a third type of city in which all workers are hybrid. Even in the largest cities, some workers will continue to work at work every day because there is a residential region that is close enough to the city centre that commuting costs do not justify sometimes working from home.

So, the model allows for two types of cities: (i) small towns which are initially unaffected by the WFH revolution because commuting is not sufficeintly burdensome for its firms and workers to benefit from working at home; and (ii) larger cities that host a mixture of firms with empoloyees who commute every day and firms with employees who sometimes WFH. Nonetheless, these larger cities that host hybrid workers and firms are also not equal. The distance thresholds are functions of the opportunity cost to productivity and wages found in equation 18, $\theta_h = \frac{d_h}{d_o + d_h}\overline{a}\theta_o$. The opportunity cost to wages θ_h is a function of commuting distance, but the opportunity cost in terms of lost productivity per day, θ_o, is a parameter of the model. So, larger cities with more workers commuting over longer distances enable a larger share of workers to WFH and those workers will commute less frequently thain hybrid workers in smaller cities. Consequently, larger cities experience a greater decline in overall output and observed productive efficiency. But the WFH revolution also offers a greater welfare gain and greater profitability in larger cities than in smaller cities, attracting workers and firms to relocate and creating a shadow effect over smaller cities as the system returns to a spatial equilibrium.

5.3 What if offices for hybrid workers become smaller?

There is also a possibility of an alternate scenario if the office-space per worker parameter g is a function of commuting frequency. Firms with workers who regularly WFH may not require dedicated work spaces, or may require different types of work-spaces for in-person meetings at work because desk work occurs at home. Since g is the denominator in the slope function found in equation 14, the reduction in the size of the office space for hybrid workers steepens its bid-rent curve. If this effect is greater than the flattening of the of the bid-rent curve due to reductions in commuting frequency the bid-rent curve for office space for hybrid workers may even be steeper than the bid rent curve for office space for workers who work at work. This would push firms with employees who commute more frequently to locate in more distant locations where office space is cheaper, in order to provide sufficient office space for their in-person workers at a sufficiently low cost. Meanwhile firms with employees who sometimes WFH are pulled into the city centre, because they don't need much office space for them at all. This generates two further types of cities after the WFH revolution.

Reconsider the slope of the bid-rent curve for office space found in equation 14, with office-space per worker as a function of commuting frequency. For a firm with workers who commute every day the slope is

$$\frac{\partial r\left(d_o\right)}{\partial d_o}=-\frac{\phi f_{max}^n}{g\left(f^*=f_{max}\right)}, \tag{47}$$

whereas the slope of the bid-rent curve for office space for firms with workers who WAW is

$$\frac{\partial r\left(d_o\right)}{\partial d_o}=-\frac{\phi f^{*n}}{g\left(f^*\left(d_o\right)\right)}. \tag{48}$$

Rearranging the inequality condition for this type of city in which hybrid firms have a steeper bid rent curve than firms with workers who always WAW, gives

$$\frac{g\left(f^*\left(d_o\right)\right)}{g\left(f^*=f_{max}\right)}<\frac{f^{*n}}{f_{max}^n}. \tag{49}$$

That is, the ratio of office space required in the commercial district for workers who regularly WFH compared to workers who always WAW must be smaller than the ratio of commuting frequencies. Put simply, if the decline in office space requirements for hybrid WFH is greater than the decline in commuting frequency, hybrid offices will sort into locations closer to the city centre with commuting frequency increasing with distance.

In these cities there is a reduction in the size of the commercial district because there is less demand for office space when hybrid workers do not require as much. The sorting of hybrid firms closer to the city centre mitigates this effect to some extent since hybrid firms close to the city centre require more frequent commuting than if they were further away. This implies an inverse donut effect within the office district due to changes in slope. Rather than a flattening of the slope of the bid-rent curve, frequency optimization steepens the slope of the bid-rent curve for office space, which mitigates the decline in demand, with the large declines in rent and density in more distant locations, still within the office district. In the residential district, the standard donut effect remains in which workers who live further from the city commute less frequently. The emergence of such a city requires such a high productivity level for in person work by workers who usually WAW that there are such high rents in the city centre that only those with the most valuable activities can afford to pay such rents. Renting such office space may be periodic, such as renting office facilities on demand or utilizing a co-working space.

The condition given in equation 46 still applies. So, when that threshold is reached, all firms would switch to allow workers to sometimes WFH. As a result, such cities would host no firms in which all work occurs in person.

5.4 Shadow effects in a system of cities

While all workers are initially either better off or no worse off after the WFH revolution, the change in welfare is uneven across the city and in cities of different sizes because f^* differs for each pair of work and home locations. Similarly, while the marginal cost of production declines for any firm in the city's commercial district with workers who choose to sometimes WFH because they avoid their share of commuting costs, the change in marginal cost is uneven across the city and in cities of different sizes. While our conceptual model considered two non-continguous cities in Section 2, and our detailed model considered a single closed city in Section 4, real world location decisions occur within a *system* of cities in which spatial equilibrium equalizes utility both within each city and across all cities in the system. With uneven changes in utility across space, the spatial distribution of homes and offices that existed prior to the WFH revolution—both within and between cities—is no longer a spatial equilibrium. Returning to spatial equilibrium involves two types of changes with ambiguous effects on city structures and on different types of cities.

Each city is characterized by its central reference point for commuting. Location decisions in a system of cities are made in relation to *all* cities in the system. That is, workers and firms choose between the reference points of all city, and a location within the city they select, that maximises welfare (and consequently rent) and profit, respectively, with welfare equalizing within and between cities in spatial equilibrium. The relevant reference point for each worker is the city that achieves their highest level of utility by accepting employment at a firm in that city. Furthermore, bid-rent curves can overlap between cities, in which two alternate reference points both have positive bid-rent curves in the same location, such that neighboring residents may even commute to different city centres.

The flattening of bid-rent curves could result in a new overlap, or an increase in the size of the overlap between cities. Even without migrating between cities, workers (and firms) can "relocate" by changing the reference point to another city if it offers higher utility. With uneven changes in utility across space, it is possible that workers located towards the edges of some cities, now in regions with overlapping positive bid-rent curves, may switch reference point to a nearby city that now offers a higher level of utility after the WFH revolution, without changing the location of their home. While previously commuting a long distance to a nearby large city deterred such workers from accepting employment at a firm in that city, the ability to commute less than every day reduces that deterrrent, possibly to the point of switching to an employer in the larger city without changing the location of their home.

Secondly, workers may choose to migrate to another city. Prior to the WFH revolution, long commutes are a significant deterrent to living in a large city. But if workers do not have to commute every day, then commuting is less of a deterrent. With frequency optimization, the decline in commuting frequency is greatest for longer commutes, and so the increase in welfare is greater in a larger city. As a result, workers are incentivised to migrate to larger cities where they have to accept employment that involves longer commuting distances, but they are willing to do so in spite of the long distance commute because they do not have to commute every day.

As the system of cities returns to spatial equilibrium, both types of relocations generate a shadow effect, such that economic activity declines in some places and expands in others. Furthermore, the reduction in commuting costs (effectively a transaction cost for work) increases the overall productivity of the system with the greatest increases in the largedst cities. While the WFH revolution is initially welfare enhancing everywhere, the ultimate net effect of such changes may not be.

Changing reference points

The model developed in section 4 is based on a reference point in the city centre. A system of cities involves multiple references points in which each represents the centre of a separate town or city and workers and firms can choose where to locate (and relocate after the WFH reolution) in order to reach a spatial equilibrium. Even prior to relocation, the relevant reference point for an individual can change because the spreading out of bid-rent curves in larger cities, due to its flattening, may now overlap with smaller cities. Then, if the larger city offers a higher welfare or profit, the relevant local reference point for those smaller cities changes to the larger city, even before any relocation decisions occur. Since such relocation decisions are based on specific systems of cities and their distances from one another we do not estimate a simulation for any particular arbitrary system. Instead we consider this intuitively based on various potential geographies.

The expansion outward of larger cities offers residents in small nearby towns, the opportunity to join the hinterlands of these larger cities. Workers living in this potential hinterland can now offer their labour in the expanded labour market of commutable distances that include the nearby larger city. This hypothetical choice facing firms and workers means that the larger city absorbs parts of, if not all, the smaller towns in its hinterland, into the larger cities bid-rent, density, and other curves that describe the spatial distribution of economic activity.

Each firm-worker pair in the expanded hinterland, is now also subsumed into a nearby larger city, with a reference point in the centre of the larger city. This captures the fact that local businesses can now access a larger labour market pool of potential employees and workers now have a access to a larger pool of employers in the larger city and its hinterland combined. The relevant thresholds apply as above, but the population of the larger city now includes its surrounding hinterlands, and the reference point to those thresholds is determined relative to the larger city, even if these two places previously operated as separate labour markets. Where a hinterland town is subsumed into a larger city, the overall effect on residents in the urban hinterland is positive, and could be substantially positive since switching the reference point to the larger city establishes a significantly higher productivity for firms and workers. If the welfare effect were negative, then workers would continue referencing the local centre and the hinterland would not be initially impacted by its larger neighbor.

Where a smaller city is only partially subsumed by the hinterlands of a larger city, some of the small city's workers switch to employment in the larger city. As a result, the small city office district where those workers were previously employed reduces in size, creating a sh

But firms in this hinterland may now be outside the commercial district and will have an incentive to either relocate to the commercial district of the larger

Migration between cities and shadow effects

The intuitive response to the WFH revolution, especially from policy-makers, is that this is an opportunity for the most remote places to attract new residents and job opportunities and that perhaps the WFH revolution will spark a revival of development in the distant or isolated left-behind places. Intuitively this makes sense on the basis of a closed city model with a singular central point around which all economic activity is organized because more distant locations achieve a greater increase in utility. But in a system of cities, the unequal change in utility incentivises workers to relocate to other cities with higher utility, untill the population distribution returns to a spatial equilibrium. Rent for homes and population density will increase in locations with relatively higher utility while the source locations of relocating workers would reduce in price and population density, untill all workers achieve the same increase in utility in spatial equilibrium. Similarly, the unequal change in the marginal cost of production for firms hosted in different locations incentivises firms to relocate to places where marginal costs are lower than their former host location, untill the joint distribution of firms and workers returns to a spatial equilibrium. Rent for office space and the density of firms will increase in locations now offering the opportunity to produce with relatively lower marginal costs while the source locations of relocating firms would reduce price and density untill all firms achieve the same decline in marginal cost in spatial equilibrium.

Therefore, cities where the initial utility increase (prior to subsequent relocation decisions and rent price changes) is equal to the distance-based *average*[10] increase in utility would observe no population change and places where the decline in marginal cost is equal to the *average* change in marginal cost would observe no change in the density of firms. But shadow effects would appear in any place with an increase in utility that is less than average or a decrease in marginal costs that is less than average, as the changes are smaller than more optimal locations where the increase in utility or decline in marginal cost is greater than average. Equivalently, places with greater than average changes would experience population increases, rent increases, and density increases. Nonetheless, after the model returns to spatial equilibrium, all workers are equally better off and all firms have equally lower marginal costs.

Setting the change in utility in equation 41 to its average $\overline{\triangle U}$ and solving for distance implicitly defines the boundaries of the residential region of the city at which there are neither shadow effects or population increases. Between these boundaries there are population increases and beyond these boundaries there are shadow effects. The definitions are implicit, since the optimal commuting frequency is also defined by a location. Just as there are two locations that define where the change in utility is zero, similarly, there is a smaller bounded region between two locations where there is a greater than average change in utility. Places experience shadow effects if they are closer to the city centre than the inner boundary of this region or beyond the outer boundary.

To define this region more clearly, consider that the change in utility, prior to any relocations taking place is $\triangle U = U^* - U = -\triangle C$ where the star indicates that the variable is determined by optimal commuting frequency behavior after the WFH revolution and $C = \phi d_h f^n + \theta_h f^{-m}$ is the overall commuting cost component of the budget constraint in equation 8. The change in overall commuting costs is

$$\triangle C = \phi d_h \left({f^*}^n - f^n_{max} \right) + \theta_h {f^*}^{-m} \tag{50}$$

where f^* is the optimal commuting frequency at d_h shown in equation 16. Setting this to its average $\overline{\triangle C}$ and solving for distance gives the implicit definition for the distance from the city centre where there are neither shadow effects on workers or population increases

$$d_h = \frac{\overline{\triangle C} - \theta_h . {f^*}^{-m}}{\phi \left(f^{*n} - f^n_{max} \right)}. \tag{51}$$

The definition is implicit, since f^* is also a function of distance. Beyond this boundary there are population increases as workers seek lower housing costs by taking advantage of the opportunity to commute less frequently when they live further away. In the normal case where $n = m = 1$, the explicit definition is

$$d_h = \frac{\overline{\triangle C} - \theta_h}{\phi \left(1 - f^n_{max} \right)}. \tag{52}$$

Places closer to the city centre than this threshold will have lower than average changes in overall commuting costs and experience shadow effects, while places between the threshold will experience population increases. Similarly, the average change in the marginal cost of production is

$$\triangle MC = \left(\phi.d_o.\left(f^{*^n} - f^n_{max}\right) + \left(\overline{a}\theta_o - \theta_h\right)\left(f^{*^{-m}}\right)\right)\frac{1}{\overline{a}}. \tag{53}$$

Setting the change in marginal cost to its average and solving for distance implicitly defines the boundaries of the city at which there are neither shadow effects on firms or increases in firm density,

$$d_o = \frac{\overline{a}\overline{\triangle MC} - \left(\overline{a}\theta_o - \theta_h\right)\left(f^{*^{-m}}\right)}{\phi.\left(f^{*^n} - f^n_{max}\right)}. \tag{54}$$

In the normal case ($n = m = 1$), and substituting firm preferences for commutnig frequency in Equation 16, the explicit distance is

$$d_o = \frac{\overline{a}\overline{\triangle MC} - \left(\overline{a}\theta_o - \theta_h\right)}{\phi\left(1 - f^n_{max}\right)}. \tag{55}$$

While at first glance both of these calculations appear to be describing the donut effect in city centers, the shadow is contingent on the magnitude of the *system-wide average* change in utility and marginal cost whereas the shadow area in the donut effect is a within city phenomena. This means that the scale of the inter-urban shadow area in each city does not differ between cities of different sizes. As a result, some smaller cities could be entirely within the shadow region resulting in a city-wide shadow effect. In the largest cities, this inter-urban shadow region could be smaller than the intra-urban shadow region from the donut effect, meaning that larger than average utility gains in the region just beyond the inter-urban shadow effect would partially mitigate the city's own donut effect. This inter-urban shadow effect captures the effect we sketched in Figure 3.

Of course, the relevant locations where shadow effects occur would have to be determined on a system by system basis, since the average change in utility and marginal cost varies between systems, by calibrating the equations to real world rents, population densities, commuting frequencies, and transport costs. Nonetheless, we can intuitively consider how different types of systems (e.g. countries) would be affected. Systems of cities with population distributions skewed towards the largest city could expect to see greater shadow effects across their smaller cities, which are unable to offer as great an increase in utility, and the dominance of the largest city would be further enhanced by the WFH revolution. Systems with populations more evenly distributed in cities of similar sizes would have smaller shadow effects, if any, but would experience smaller increases in welfare overall than systems with uneven city sizes. Systems with only small cities might not experience much of a change at all in terms of welfare gains or shadow effects, but could still take advantage of WFH by increasing the extent of labour market pooling.[11]

As a result of these effects, a return to spatial equilibrium generates migration towards, and employment switching to, larger cities—which is already observed in recent data (Mondragon and Wieland, 2022)—and greater opportunities to WFH in these larger cities—concurring with with recent empirical evidence from the American Community Survey that commuting zones with longer commuting times experienced higher rates of remote work (Ozimek and Carlson, 2022). This implies increases in economic activity in larger metropolitan areas *and their hinterlands* where utility and disposable incomes have increased by greater amounts than in smaller cities, since commuting is now less burdensome and the opportunity cost of working from home is less than these commuting cost savings. That is, both workers and employers move to larger cities and their hinterlands where there are greater benefits from hybrid WFH opportunities. These conclusions are only drawn from our theoretical model because it allows for workers and firms to optimise commuting frequency alongside location decisions. Other models that directly calibrate working from home behavior, tend to predict only the donut effect and suggest workers move away from larger cities (Delventhal et al., 2022; Kyriakopoulou and Picard, 2023; Gokan et al., 2022; Delventhal and Parkhomenko, 2023; Brueckner et al., 2023; Behrens et al., 2024; Davis et al., 2024). Rather than allowing for work-from-anywhere that generates economic development in more isolated places, our model shows that the WFH revolution implies potential shadow effects on smaller cities that are too distant to host many commuting hybrid WFH workers or too small to offer substantial benefits from hosting the employers of hybrid WFH workers.

6 Discussion and Conclusions

The explicit inclusion of location and commuting frequency optimisation behaviour provides a much more nuanced description of how the WFH revolution affects economic activity, both at the intra-urban intra-regional scales, as well as at the inter-urban inter-regional scale. Commuting behaviour is a trade-off between marginal productivity of commuting frequency and location that will vary considerably by industry and location. The key results which emerge from this analysis are that in response to the WFH- 'zoomshock' revolution, the intra-urban rent gradient (usually) flattens, giving rise to the so-called 'donut' effect, and also that this flattening favours the larger cities, giving rise to an inter-urban 'shadow' (Cuberes et al., 2021) effect. In other words, the joint effect depicted in Figures 1 to 3 holds. While the first 'donut' result has already been widely observed, this second 'shadow' effect has not before been understood. Indeed, the fact that the shadow effect favours larger cities and their hinterlands is a result which appears to be counterintuitive to most commentaries and policy initiatives on the economic implications of the 'zoomshock', which have tended to emphasise the potential economic development possibilities for smaller and more remote places. However, our result shows that the WFH revolution implies that falls in commuting frequency favour larger places where commuting distances are longer, and a kind of shadow effect on small towns and cities that do not really benefit much at all from the opportunity to commute less frequently. This latter 'shadow' effect has the potential to generate major changes in economic geography, especially in places where cities are in proximity with each other such that commuting hinterlands abut, as well as perpetuating the ongoing relative decline of left-behind places in more distant and rural regions.

Under the recent widespread shift to increased WFH as a result of the 'zoomshock', the new trade-off provides the greatest benefit to people and firms located in large cities and especially their hinterlands because these are the locations where commuting is most burdensome. The 'donut' effect arises because for any job where at least some tasks offer higher productivity with face-to-face interaction there will be an optimal commuting frequency for hybrid WFH that allows workers to live further from their workplace, and allows workplaces to locate further from city centres. At the same time, this same trade-off gives rise to a 'shadow' effect, because commuting is a significant burden of living in large metro areas and the shift to fully- or partially- remote work reduces that burden by a greater amount in places with longer commutes resulting in a greater welfare gain in large cities and for people who relocate to large cities. Commuting is now less of a deterrent to locating in a large

Table 1: Percentage Change 2019-2022 Pre- to Post-Covid Lockdown

	$\triangle f^*$	$\triangle d$	$\triangle t$	$\triangle D = \triangle (f \times d)$	$\triangle T = \triangle (f \times t)$
Urban Conurbations	-21.2	+2	-5.7	-16.9	-24.7
Urban City & Town areas	-18.75	-12	-7.4	-7.3	-19.3
Rural Town & Fringe areas	-2.5	-4.9	-14.2	-7.3	-19.3
Rural Villages, Hamlets, & Isolated Dwellings	-6.7	-1.6	No change	-8.3	-7.4
London	-25.4	+15.6	-2.3	-13.8	-26.3
England	-16.2	-5.8	-9.7	-21	-23.2
England excluding London	-13.9	-9.8	-7.1	-22.4	-21.8

Source: National Travel Surveys 2019-2022

city, so both firms and workers face a stronger attraction to relocate to larger cities, or at least their hinterlands, which used to have a costly commute. As such, the WFH revolution generates more inclusive opportunities for productivity growth in the regions surrounding the largest cities, but implies shadow effects or economic decline in other places that are less suited to WFH.

Evidence to support these parallel predictions comes from the National Travel Surveys of England. For our purposes, and using our model notation, the key variables of interest are as follows: f = average trip frequency per commuter; d = average trip distance per commuter; t = average trip time per commuter; $D = f \times d$ = average total annual commuting distance per commuter; $T = f \times t$ = average total annual commuting time per commuter.

Table 1 outlines the broad England-wide aggregate changes in each of these key variables 2019-2022. What we see from the Table 1 above, which is constructed from the NTS National Travel Surveys 2019-2022, is that increased WFH-hybrid work is, as expected, reducing average commuting trip frequencies in all types of places, as well as the resulting total annual commuting time and commuting distances covered. Yet, while employment-commuting frequencies fall in all places, it is only in the largest conurbations that average commuting distances increase. Commuting distances actually fall in all other types of urban areas, including smaller cities, towns and scattered settlements. Interpreting the data alongside the model implies that English cities fall neatly into the two categories of cities found in our model, and perhaps a third rural category that we did not model. Neither this analytical result nor this empirical result have been observed and accounted for before by previous frameworks, whereas these simultaneous results are exactly in line with our model.[12]

Rather than allowing work from anywhere, the work-from-home revolution therefore generates greater forces to live within a commutable distance of ever-larger cities, creating shadow effects on smaller distant cities. This shadow effect therefore provides a new twist to the urban bias growth already evident in many places (Eckert et al., 2022; Finlay and Williams, 2022; Giannone, 2022) and implies potentially further regional divergence (Ganong and Shoag, 2017).

Of course this model does not cover *all* of the impacts of the WFH revolution. When workers spend a larger share of time at home, their demand also shifts location, stimulating demand for local services. An extension of the model could accomodate this by including a spatial transaction cost for some consumption goods. This would shift the location of some commercial activities into the residential area when workers WFH. There would be a decline in economic activity in city centres and an increase in residential areas, but the real world gain in the suburbs is probably smaller than any loss of activity in the CBD since food-at-home is a much closer substitute to food-service in the suburbs than homemade-food-in-the-office is a substitute for food-service in the city. A related point is that WFH is more flexible in terms of *when* commuting occurs, so there could be a smoothing of these types of service demands over time. That is, if people shift demand from weekends to weekdays and from evenings to day-time, this could reduce employment in some service sectors and generate productivity benefits by shifting demand away from peak periods. Studying both of these phenomena is proably better explored with other models, but conclusions such as these can be intuitively considered within our framework. In light of the findings in this paper, the such growth in suburban economic activity is likely to be greatest in the suburbs and hinterlands of larger cities.

Arguably, these forces favoring larger cities could be counteracted if some of the productivity benefits of agglomeration economies are now shared more widely by smaller cities able to access remote work tools. Simply treating the WFH or WAW as a binary choice lessons the urban wage premium (Liu and Su, 2022). However, our analysis which makes the commuting frequency the key choice variables implies that this is only relevant to the extent that the decrease in relative productivity between small and large cities is greater than the savings from reduced commuting by working remotely in the larger city. Given these technologies are designed for WFH, not WAW in smaller cities, it is also not at all clear that this will be the case. While some of the former agglomeration benefits may now be accessible anywhere, it is only for jobs that become fully-remote that would imply activity shifts away from the larger cities and their hinterlands. Furthermore, fully-remote work may be gradually out-sourced overseas in much the same way that it already is. Nonetheless, the largest cities also offer amenities so the reduced burden of commuting would still imply that even fully-remote workers face a greater increase in the attractiveness of the hinterlands of large cities than rural areas. This means that the only rural areas that would would truly benefit from fully-remote work would be those places with significant *natural* amenities that could not previously host those activities. As yet, there is no evidence that the WFH-'zoomshock' revolution has spurred innovation and productivity growth in general (The Economist, 2022). However, our analysis implies that the returns to productivity growth will be reconfigured spatially, and this reconfiguring is likely to benefit larger and more prosperous cities than smaller or less prosperous places.

Notes

[1]This approach is different to the standard urban economics approach which employs the iceberg costs construction (Behrens et al., 2024). In the case where distance is an explicit consideration the iceberg structure is inconsistent with typically observed transport rate structures (McCann, 1995), all of which vary with respect to the square root of distance (McCann, 2001; Bosker and Garretsen, 2010) precisely because of these frequency-optimisation issues (McCann, 2001). Furthermore, while a strong assumption in urban economics is that transport costs are directly associated with the wage rate, as reflecting the opportunity costs of time, the empirical evidence suggest that these account for no more than 15% of commuting costs (Van Ommeren and Dargay, 2006). Moreover, the use of modern internet-based technologies means that commuting nowadays often involves working-on-the-move, such that the time-related opportunity costs are further reduced, in some cases to close to zero. To a large extent commuting also takes place primarily out-of-office hours, and accounts for no more than 15%-20% of all trips (Tomer et al., 2020). These various observations combine to weaken, or even undermine, the empirical veracity of the simple iceberg assumption in the context of employment-commuting frequency choices, unless the iceberg stricture itself is

complemented by an additional distance-frequency related features, as is done here.

[2]For businesses, commercial face-to-face interactions can take place multiple times a day between different personnel, and indeed, this is exactly how many high value knowledge-intensive and business services and retailing activities work. In principle, the maximum value of f for such businesses can be in terms of many thousands of such interactions per year. For commuting workers, the situation is different. Each worker will typically commute to and from work once per day. In terms of the number of working days, a typical year has 261 working days. Adding public holidays to paid leave allowances mean that a typical US worker will annually work approximately 235 days, while across the OECD the typical number of working days per year are between 225 and 230. These values represent the maximum value of f for commuters, which is much lower than for business interactions, but the analytical principles remain largely the same. This simple model uses the assumption that $f \to \infty$. However, in the richer model below, we assume that $f \to f_{max}$.

[3]The exact functional form of g is likely to require additional microfoundations that include the coordination of remote work on different days, the share of workers who can hot-desk and the share of in-person tasks that are team-based, or the type of office space required for in-person tasks. We leave this for future research.

[4]That the existence of hybrid remote work reveals that the marginal productivity of WFH is less than WAW can be explained as follows. The change in output and wages (θ_o and $\overline{a}.\theta_h$ respectively) due to *marginal* productivity differences for WAW compared to WFH will likely be a function of the frequency of commuting. While we do not examine the functional form of output and wages relative to commuting frequency, if the productivity of WFH were diminishing at a higher rate than WAW, standard optimisation where marginal benefits equal marginal costs implies that the frequency of commuting reduces untill the marginal cost saving from less frequent commuting is equal to the marginal productivity loss. In equilibrium, this is either full WAW, full WFH, or a hybrid-combination of WFH and WAW. An equilibrium with full WAW or hybrid WFH means that that the marginal productivity of WAW is higher than WFH, in order to justify the cost of commuting at least sometimes. An equilibrium with full WFH reveals that any potential increase in productivity from WAW does not justify the cost of commuting. Thus our assumption is just a simplification the outcome when an equilibrium level of commuting reveals a preference in the labour market for hybrid work.

The model could also easily accommodate the possibility that marginal productivity at work is always higher than marginal productivity of work-at-work. In this scenario, the equilibrium is that all work is conducted at home, office space would disappear entirely, and the locations of workers' homes are no longer related to firm locations. A model of location decisions would then need to consider other reasons that people commute elsewhere in the city, such as for school, visiting friends, recreation and amenities, some of which can be experienced remotely, reducing the frequency of commuting. Assuming diminishing marginal utility for in-person 'leisure', people would optimise commuting behavior and location choices in the same way as our model which would not change the fundamental insights of the model. The new insight would be that the distribution of people and economic activity would switch from one fundamentally based around commuting to work, to one that is fundamentally based on commuting for leisure. In this way, our model develops the appropriate framework for addressing such questions. We leave this for future research if such a scenario becomes realistic.

[5]See Bond-Smith (2022) for how the analysis could be adapted to a smaller discrete number of firms, though it requires a more complex specification with constant elasticity of substitution and fixed costs.

[6]It is straightforward to adapt the model to monopolistic competition with increasing returns to scale in which firms face a fixed cost and prices are the marginal cost multiplied by a constant $\sigma/(\sigma-1)$ where σ is the constant elasticity of substitution. It does not change the thesis of the article, but would allow other extensions such as examining how WFH affects growth via innovation in an endogenous growth model. See Bond-Smith (2024) for an example of how endogenous growth may be added to an AMM urban model.

[7]Land price must have some positive value, otherwise quantity of housing tends to infinity in our specification.

[8]Alternatively, we could assume that θ_o is sufficiently high (which also applies to θ_h) that f^* equals f_{max} prior to the WFH revolution. However, it would require making an additional assumption about the extent that θ_o declined with the improvement of telecommuting software and other business practices. There is no doubt that such a decline also occurred, but the approach we use here here does not rely on any assumptions about the extent of such a decline.

[9]Prior to any relocation decisions occurring, there is no change in rents, but we assume that all other changes occur instantaneously, even if in practice all changes will take some time to reach an equilibrium

state. For example, the wage decline, θ_h is treated as a parameter of the model that interacts with the frequency of commuting, in order to understand relocation forces, but obviously this adjustment will also take some time.

[10]All subsequent references to average utility are distance based, not population based.

[11]This particular extension would require a heterogeneous labour force.

[12]These results are generated purely on the basis of rapid post-covid-19 shifts in commuting behaviour, and are not dependent on other assumptions regarding possible longer-run changes in agglomeration-density relationships in potentially shrinking city-centres (Kyriakopoulou and Picard, 2023). Nor can they be ascribed to larger urban areas having a larger share of skill workers with jobs are most amenable to homeworking, as this is not the case in the UK, except for London. Many Urban City and Towns, along with Rural Town and Fringe areas in the UK display higher shares of such workers than the Conurbations (Rodrigues et al., 2022).

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